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LETTER TO THE EDITORS

COMMENTS ON THE PAPER "THE SIMILARITY HYPOTHESIS APPLIED TO TURBULENT FLOW IN AN ANNULUS" BY H. BARROW, Y. LEE AND A. ROBERTS

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IN THEIR RECENT PAPER [1], the method by which Barrow, Lee and Roberts attempt to apply the similarity hypothesis seems to this writer to be unsound in concept.

They propose that the application in turn of their equation (1) to the inner and outer regions of the annulus should result in expressions for the velocity defect in these regions which are independent of the annular radius ratio. So that if the abscissa of the graph on which either of these expressions is plotted has a given value then the velocity has only one corresponding value for all radius ratios.

If this were so, it would be reasonable to expect the expressions to describe correctly the profile in an annulus whose radius ratio was as arbitrarily close to unity as desirable; that is an annulus which was effectively a parallel-plate passage, in which, of course, the inner- and outer-region velocity profiles would be identical, as must be the velocity defect expressions.

Thus it seems to this writer that the postulate of radius-ratio independence is contrary to the postulate of different velocity defect expressions for the inner and outer regions [Figs. 2(a) and 2(b)].

Further, the authors argue at some length that the poor agreement of their equation (11) with their experiments in the inner region is due to the assumption concerning the turbulence characteristics not being valid in the inner region. It is noteworthy that neither does it agree with the experimental results quoted by Goldstein ([2], p. 352) for channel flow where this assumption is at least as valid as it is in the outer region. According to the authors' postulate of radius-ratio independence of their velocity defect expressions, their equation (11) should agree if the assumption concerning turbulence characteristics is valid.

It is perhaps pertinent to this observation to note that the authors' equation 1 (ii) is stated by them to be an expression for the shear stress, whilst equation 1 (iii) is used to derive to velocity defect expressions.

Goldstein ([2], p. 354) states explicitly that equations 1 (ii) and (iii) are to be regarded as equal possibilities from which velocity expressions can be derived; the first leading to an equation of motion similar to Prandtl's momentum theory, and the second, to an equation similar to that from Taylor's vorticity transfer theory with symmetrical turbulence. The second, used by the authors, is shown to give poor agreement with channel flow.

Finally it could be argued that the authors' approach is justified in a heuristic manner by considering the agreement of their theory and experiments in Fig. 2(a). However the experimental results of Brighton and Jones [3] in the outer region show a notable effect of radius ratio and a very pronounced effect of Reynolds number.

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AUTHORS' REPLY

WE ARE pleased to have Mr. A. Quarmby's comments on our paper, and to learn that he has given careful consideration to its contents.

In our paper, we have derived expressions for the velocity defects in the inner and outer regions of an axisymmetric turbulent annular flow, employing Goldstein's [1] expressions for l and M (viz. equation 1 (i) and 1 (iii) in the paper). As stated, equation 1 (iii) was chosen in preference to 1 (ii), which is an expression for the shearing stress, because in the case of axisymmetric pipe flow it leads to a better result ([1], p. 494). This appeared to be a logical choice for the study of the annular flow in view of the fact that the pipe geometry can be considered as a particular case of the annular geometry. At least, the flow in both geometries is axisymmetric. At the outset, a postulate concerning the independence of the radius ratio is not made. Indeed, no such condition is essential to the derivation of the final expressions for the velocity defect as an examination of our analysis will show. Cognizance of the radius ratio is taken in arriving at equations (9) and (11) by the use of equation (5) and Lamb's expression for r_m. It is, of course, true that once the constants k and b have been assigned values, the velocity defect for a given dimensionless wall distance is, according to the theory, the same for all radius ratios. (It is to be noted that the velocity defects are defined in terms of the appropriate boundary stresses, the ratio of which is a function of the radius ratio). Considering, however, the actual velocity defects, $(u_m - u)$, at given wall distances, it will be seen that the ratio of these defects is a function of radius ratio.

For the outer region, the independence of α is to be considered as an experimentally determined fact, and is not a postulate in the theory which has its origin in equations 1 (i) and 1 (iii). Close scrutiny of Fig. 2 (a) shows that there is a small variation with α, but this could possibly be due to the radius of maximum velocity being different from Lamb's radius, particularly for the case of $\alpha = 80{\cdot}72$. Another factor which must also be considered when accounting for the differences is the distribution of turbulence characteristics. Different annuli exhibit different distributions.

Some explanations of the systematic departure of the data for the inner region from equation (11) has been given. Attention has been drawn to the turbulence characteristics because their distribution is vital in the theory. Because the distribution in the inner region does not approximate to the mathematical model, one might expect less success in correlation in that region. With regard to the Reynolds-number effect, it is clearly seen in Fig. 2 (a) that there is no systematic variation of the velocity defect with Re.

Quarmby's point on the velocity defect in the annular geometry warrants further discussion, and in this connection it is interesting to examine Brighton's [2] work. Brighton studied turbulent flow in four annuli with $\alpha = 1{\cdot}78, 2{\cdot}66, 8$ and 16. At the outset of his comparison of results, he considers the use of a single velocity defect law in the form

$$\frac{u_m - u}{u_\tau} = f(y/\delta) \qquad \text{(A-1)}$$

Brighton found that he was able to correlate his data for the outer region by a single form of equation (A-1), but that the data for the inner region showed increasing differences with increasing α. There was no marked dependence on Re for the outer region. His results are then similar to ours, and it will be found that his equation is almost the same as equation (9).

The second point raised by Quarmby is really concerned with the extrapolation of the results which are derived for an axisymmetric flow model to the case of two dimensional flow. The parallel channel might be considered as a limiting case of the annulus, but the flow model is different and it is necessary to derive the velocity defect anew using expressions equivalent to those in equation (1). This case is considered by Goldstein [1] as a separate problem and the distinction is made. For the channel, the two dimensional forms of equation (1) give the better result. The present theory is concerned with axisymmetric flow, and extrapolation to two dimensions must be exercised with caution. It is interesting to note that the present result is compatible with that for the pipe. Extrapolation to the limiting annulus, i.e. the channel, must not be made without question because of the premise on which the theory is based. The present writers are then of the opinion that Quarmby's concern for limiting case of the parallel channel is not completely justified.

Concerning his final remark on the experimental results of Brighton and Jones there is clearly no systematic dependence on the Reynolds number.

REFERENCES

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